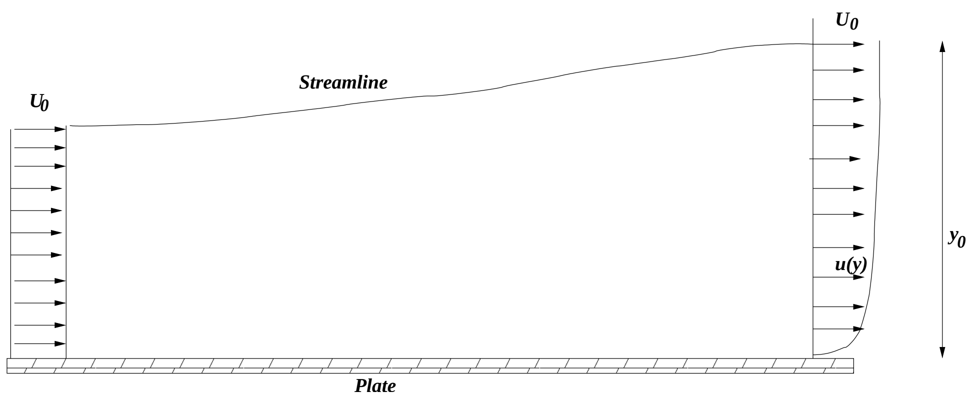
SIO 214a Introduction to Fluids Problem Set 3
Due Monday, 15 October 2007

1. Water flows out of a $1\ cm$ diameter faucet at a rate of $0.1\ Kg\ s^{-1}$. Assuming that the vertical velocity is the same across any section in the water stream, and that the pressure within the stream is equal to the pressure at the interface between the stream and the air, determine the velocity of the water when it reaches the bottom of the basin, $20\ cm$ beneath the faucet.



2. Fluid of uniform velocity U_0 and constant pressure flows over a flat plate. Due to the action of viscosity, the fluid adjacent to the plate is slowed down and at the right end of the plate the fluid inside the *boundary layer*, where $y \leq y_0$, varies so that $u = U_0 f(y/y_0$. Show that the drag force (the force directed parallel to the plate) per unit width is

$$D = \int_0^{y_0} \rho(U_0 - u)u\ dy$$

3. An idealized estuary is modeled as a box of constant depth H, constant width W and length$L$ extending inland from the ocean. The x-axis starts at the open end of the box and extends inland. The z-axis is located at the time averaged position of the free surface. The tide forces the free surface to oscillate about its mean position. At any time, the free surface is located at $z = \eta(t)$. Use the princple of global mass conservation to determine the axial velocity $u(x)$, assuming it does not depend on lateral position or depth.

4. A jet of water with a diameter of $0.08\ m$ and a speed of $25\ m/s$ impinges normally on a large stationary flat plate. Find the force required to hold the plate stationary.

5. A common occurrence in the flow of a fluid is the hydraulic jump, a condition in which the water depth undergoes a sudden transition. The change in water depth is usually accompanied by obvious turbulence (white water) and can either be progressive or standing. The most obvious example of a progressive hydraulic jump is the runup of broken waves on a beach. Standing hydraulic jumps are sometimes found behind dams. At any rate, a progressive hydraulic jump can always be viewed as standing if one rides on a coordinate system fixed with respect to the jump.

   - If we imagine this to be the case and have measured the depths $h_1\ and\ h_2$ on either side of the jump, find the velocities $u_1\ and\ u_2$ by a proper application of mass conservation and the momentum theorem (assume no friction at the bottom).

- Calculate the change in mechanical energy $[\rho(u^2/2 + gh)]$ which a surface element undergoes in passing through the jump. If non-zero, discuss what happens to the loss or excess of energy and whether this places any constraints on the flow.

6. Show that the thrust developed by a stationary rocket motor is

$$F = \rho A U^2 + A(p - p_{atm})$$

where p_{atm} is the atmospheric pressure, and $p,\ \rho,\ A,\ and\ U$ are respectively the pressure, density, area, and velocity of the fluid at the nozzle exit.