SIO 214a Introduction to Fluids Problem Set 6
Due Friday, 9 November 2007

1. For turbulent flow in a round pipe, von Karman suggested that outside a very thin layer near the wall where the turbulence is negligible (this is called the viscous sublayer), the mixing length l depends only on distance from the wall (i.e. $l = Kz$ where z is the coordinate perpendicular to the wall and K is a constant). Show that if the turbulent shear stress is approximately constant and equal to the wall shear, the velocity can be written as:

$$u^* = \frac{1}{K} \ln z^* + constant$$

   where $u^* = \overline{u}/u_\tau$, $z^* = zu_\tau/\nu$ and $u_\tau = \sqrt{\tau/\rho}$.

2. We consider the turbulent equivalent to Couette flow, that is one fixed plate and another, parallel to the first, moving at velocity U some distance h away from the first. We are told that very, very near the plates, in a region of thickness $h^* = \nu/U$, there is a viscous sublayer in which the flow is steady and laminar, but that outside that layer the flow is turbulent and the turbulent eddy viscosity is given by $\nu_T = 0.4U|z - h/2|$. Thinking about the momentum theory and the fact that there is no pressure gradient, what can you say about the total shear stress (laminar+turbulent) as a function of z? Is it similar or different for laminar and turbulent flow? Find $u(z)$, assuming that ν_T is always much greater than ν.

3. Repeat the analysis done in class for wind-driven flow in a closed basin for the case where the dimensional viscosity ν varies as a function of the dimensional depth h as

$$\nu = \nu_0 \frac{h}{H}$$

   where ν_0 is a constant and H is the maximum depth. Start from dimensional equations, and obtain expressions for the non-dimensional sea level and velocities as function of non-dimensional independent variables. Choose any simple distribution of depth $h = h(y)$.