SIO 214a Solution for Problem Set 3

1. (10 points) Mass flux is constant and equal to $Q = 0.1\ Kg\ s^{-1}$ Then $u_1 = 4Q/(\rho\pi d_1^2)$ The momentum theorem then states

$$Qu_1((\frac{d_2}{d_1})^2 - 1) = gh$$

from which d_2 can be evaluated, Then $u_2 = Q/(\pi d_2^2)$

2. (20 points) This is nearly the same problem as the one done in class to find the drag on a cylinder. There are two practical ways to draw the control volume: a rectangular box extending between section 1 and 2 and from the plate up to the height y_0 where the u velocity doesn't feel the effect of the plate, i.e. $u = U_0$. The other also extends between the two sections, and begins at the bottom, but the top surface is the streamline that goes through y_0 at section 2. This is the CV drawn in the problem statement. Since we essentialy used the first method to work the cylinder problem in class, I will develop the second method in the following: as usual begin by conserving mass, call y_1 the distance between the wall and the streamline at section 1. Since there is no flow across the streamline (top surface), mass conservation tells us

$$y_1 = \frac{\int_0^{y_0} u(y)dy}{U_0}$$

Then the momentum theorem tells us that the total momentum flux out of the volume is equal to the force exerted by the plate on the CV, is we assume that the pressure gradient is zero. This is because the stress τ_{yz} at the top of the streamline is zero so long as u is constant there. Then the only remaining force is the bottom stress, $-D$, exerted by the plate on the fluid, at the plate, so

$$\int_0^{y_0} u^2 dy - U_0^2 y_1 = -\frac{D}{\rho}$$

but y_1 is known from the mass concervation, and so D, the force exterd by the fluid on the plate is:

$$D = \rho \int_0^{y_0} u(U_0 - u)dy$$

3. (10 points) Consider a control volume extending from the bottom to the surface of the estuary, and from the closed end to some distance x from the open end, then use the global conservation of mass:

$$\rho(L - x)W\frac{\partial\eta}{\partial t} = \rho(H + \eta)Wu$$

From which we get:

$$u = \frac{L - x}{H + \eta}\frac{\partial\eta}{\partial t}$$

4. (10 points) The momentum theorem tells us the force exerted by the late on the fluid is the momentum flux out of the control volume in the axial direction, so:

$$-\rho u^2 A = F_x$$

So the plate feels a force equal to $\rho u^2 A$

5. (20 points) For the hydralic jump problem see Kundu pp. 233-235

6. (10 points) The result is directly from the momentum theorem.