SIO 214a Solution for Problem Set 6

1. If the turbulent shear stress is constant, then

$$Kz\frac{\partial u}{\partial z} = \tau_T = \rho u_\tau^2$$

which we can rewrites as:

$$\frac{1}{u_\tau}\frac{\partial u}{\partial z} = \frac{1}{K}\frac{\rho u_\tau}{z}$$

Integration gives:

$$\frac{u}{u_\tau} = \frac{1}{K} ln(\rho u_\tau z) + constant$$

since the constant is free, we can rewrite as given in the problem statement.

2. This is very similar to the problem above. Take the origin of the z-axis to be at the bottom plate, and consider the lower half of the region, so that $K = 0.4 * U * z$, then symmetry guaratees that $u(z = h/2) = U/2$. Then

$$\tau_T = 0.4Uz\frac{\partial u}{\partial z}$$

Integration gives:

$$\frac{u - U/2}{u_\tau} = ln(\frac{2z}{h})$$

Remember that this is not valid close to $z = 0$. To figure that out, we would have to match with a viscous sublayer.

3. The hrizontal components of the momentum equation are, in this case:

$$h^*\frac{\partial^2 u^*}{\partial z^{*2}} = \frac{\partial \eta^*}{\partial x^*}$$

and

$$h^*\frac{\partial^2 v^*}{\partial z^{*2}} = \alpha\frac{\partial \eta^*}{\partial y^*}$$

As in class, we rescale with $\tilde{z} = z^*/h^*$:

$$u^* = \eta^*_{x^*} h^* \frac{\tilde{z}^2 - 1}{2} + (\tilde{z} + 1)$$

$$v^* = \alpha\eta^*_{y^*} h^* \frac{\tilde{z}^2 - 1}{2}$$

Then the axial transport is:

$$U^* = -\eta^*_{x^*}\frac{h^{*2}}{3} + \frac{h^*}{2}$$

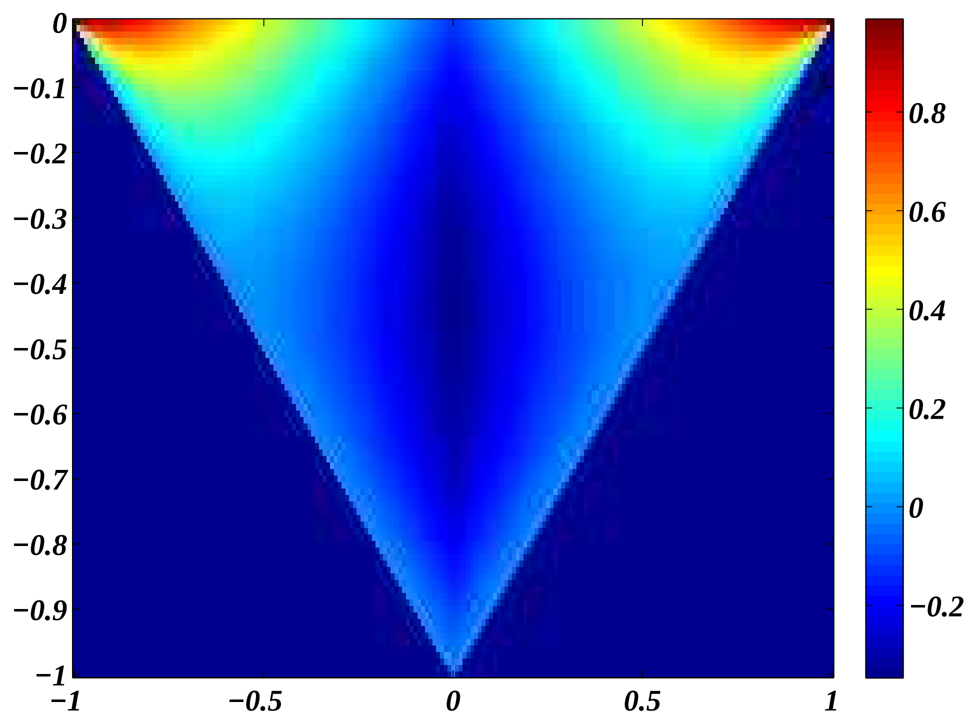
Since near mid-basin $\eta^*_{x^*}$ is constant and since the lateral integral of U^* is zero,

$$\eta^*_{x^*} = \frac{3}{2}\frac{\int_0^1 h^* dy}{\int_0^1 h^{*2} dy}$$

For a triangular basin, when $h = 1 - |y|$, $\eta^*_{x^*} = 9/4$. Then we have

$$u^* = \frac{9}{4}h^*\frac{\tilde{z}^2 - 1}{2} + (\tilde{z} + 1)$$

The solution is displayed below:



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